Schutz 6.32.

(a) The manifold being flat is easily seen from the metric being constant thus $g_{\mu\nu,\alpha\beta}$ vanish for any $\mu\nu\alpha\beta$, and Riemannian tensor vanishes.

To diagonalize $\begin{bmatrix} 0 & 1 & & \\ 1 & 0 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$ we apply

the rotation by $\frac{\pi}{2}$ to both sides to get

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \to R\left(-\frac{\pi}{2}\right)\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} R\left(\frac{\pi}{2}\right)$$

$$= \begin{pmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} \\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$$

$$= \begin{pmatrix} -1 & \\ & 1 \end{pmatrix}$$

$\Rightarrow$ g has signature of $3-1=2$.

(b) done in part (a).